**Dynamical systems**

A **dynamical system** is a concept in mathematics where a [fixed rule](http://en.wikipedia.org/wiki/Function_(mathematics)) describes the time dependence of a point in a [geometrical space](http://en.wikipedia.org/wiki/Configuration_space). Examples include the [mathematical models](http://en.wikipedia.org/wiki/Mathematical_model) that describe the swinging of a clock pendulum, the flow of water in a pipe, and the number of fish each springtime in a lake.

At any given time a dynamical system has a [*state*](http://en.wikipedia.org/wiki/State_(controls)) given by a set of [real numbers](http://en.wikipedia.org/wiki/Real_numbers) (a [vector](http://en.wikipedia.org/wiki/Vector_space)) that can be represented by a [point](http://en.wikipedia.org/wiki/Point_(geometry)) in an appropriate [*state space*](http://en.wikipedia.org/wiki/State_space) (a geometrical [manifold](http://en.wikipedia.org/wiki/Manifold)). Small changes in the state of the system create small changes in the numbers. The *evolution rule* of the dynamical system is a [fixed rule](http://en.wikipedia.org/wiki/Function_(mathematics)) that describes what future states follow from the current state. The rule is[deterministic](http://en.wikipedia.org/wiki/Deterministic_system_(mathematics)); in other words, for a given time interval only one future state follows from the current state.

There, as in other natural sciences and engineering disciplines, the evolution rule of dynamical systems is given implicitly by a relation that gives the state of the system only a short time into the future. (The relation is either a [differential equation](http://en.wikipedia.org/wiki/Differential_equation), [difference equation](http://en.wikipedia.org/wiki/Recurrence_relation) or other [time scale](http://en.wikipedia.org/wiki/Time_scale_calculus).) To determine the state for all future times requires iterating the relation many times—each advancing time a small step.

 Once the system can be solved, given an initial point it is possible to determine all its future positions, a collection of points known as a [*trajectory*](http://en.wikipedia.org/wiki/Trajectory) or [*orbit*](http://en.wikipedia.org/wiki/Orbit_(dynamics)).

Linear dynamical systems

Discrete dynamical systems

Non linear dynamical system

* <http://en.wikipedia.org/wiki/Lorenz_attractor>
* <http://en.wikipedia.org/wiki/Complex_dynamics>
  + <http://en.wikipedia.org/wiki/Julia_set>
    - <http://en.wikipedia.org/wiki/Pseudorandom_number_generator>
    - <http://en.wikipedia.org/wiki/Hardware_random_number_generator>
    - <http://en.wikipedia.org/wiki/Monte_Carlo_method>
  + <http://en.wikipedia.org/wiki/Mandelbrot_set>

Chaos theory studies the behavior of [dynamical systems](http://en.wikipedia.org/wiki/Dynamical_system) that are highly sensitive to initial conditions; an effect which is popularly referred to as the [butterfly effect](http://en.wikipedia.org/wiki/Butterfly_effect). Small differences in initial conditions (such as those due to rounding errors in numerical computation) yield widely diverging outcomes for chaotic systems, rendering long-term prediction impossible in general.[[1]](http://en.wikipedia.org/wiki/Chaos_Theory#cite_note-0) This happens even though these systems are [deterministic](http://en.wikipedia.org/wiki/Deterministic_system_(mathematics)), meaning that their future behavior is fully determined by their initial conditions, with no [random](http://en.wikipedia.org/wiki/Randomness) elements involved.[[2]](http://en.wikipedia.org/wiki/Chaos_Theory#cite_note-1) In other words, the deterministic nature of these systems does not make them predictable.[[3]](http://en.wikipedia.org/wiki/Chaos_Theory#cite_note-2)[[4]](http://en.wikipedia.org/wiki/Chaos_Theory#cite_note-WerndlCharlotte-3) This behavior is known as deterministic chaos, or simply [*chaos*](http://en.wikipedia.org/wiki/Chaos).

**Complex dynamics** is the study of [dynamical systems](http://en.wikipedia.org/wiki/Dynamical_system) defined by [iteration](http://en.wikipedia.org/wiki/Iteration) of functions on [complex number](http://en.wikipedia.org/wiki/Complex_number) spaces. **Complex analytic dynamics** is the study of the dynamics of specifically [analytic functions](http://en.wikipedia.org/wiki/Analytic_function).

In the context of [complex dynamics](http://en.wikipedia.org/wiki/Complex_dynamics), a topic of [mathematics](http://en.wikipedia.org/wiki/Mathematics), the **Julia set** and the **Fatou set** are two [complementary sets](http://en.wikipedia.org/wiki/Complement_set) defined from a [function](http://en.wikipedia.org/wiki/Function_(mathematics)). Informally, the Fatou set of the function consists of values with the property that all nearby values behave similarly under [repeated iteration](http://en.wikipedia.org/wiki/Iterated_function) of the function, and the Julia set consists of values such that an arbitrarily small perturbation can cause drastic changes in the sequence of iterated function values. Thus the behavior of the function on the Fatou set is 'regular', while on the Julia set its behavior is '[chaotic](http://en.wikipedia.org/wiki/Chaos_theory)'.

The Julia set of a function ƒ is commonly denoted *J*(ƒ), and the Fatou set is denoted *F*(ƒ).[[1]](http://en.wikipedia.org/wiki/Julia_set#cite_note-0) These sets are named after the French mathematicians[Gaston Julia](http://en.wikipedia.org/wiki/Gaston_Julia)[[2]](http://en.wikipedia.org/wiki/Julia_set#cite_note-1) and [Pierre Fatou](http://en.wikipedia.org/wiki/Pierre_Fatou),[[3]](http://en.wikipedia.org/wiki/Julia_set#cite_note-2) whose work began the study of [complex dynamics](http://en.wikipedia.org/wiki/Complex_dynamics) during the early 20th century.